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BENDING MOMENT DIAGRAMS FOR RIGID FRAMES

J. S. KENNEDY

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THESIS

Bending Moment Diagrams for Rigid Frames

Submitted as the partial fulfillment
of the
Requirements for the Degree of
Master of Science

by

James Stewart Kennedy

Under the direction of
I. F. Morrison
Professor of Applied Mechanics

University of Alberta

Edmonton, Alberta

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ABSTRACT

This work is designed to be an extention of the book, "Rigid Frame Formulas" by A. Kleinlogel. It contains the solutions for a number of the more general types of Rigid Frames in common use, for loadings consisting of couples applied at the corners of the frames. The purpose is to supplement the cases given in the book in order that its use may be extended to the solution of more complicated built-up frames.

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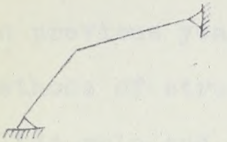
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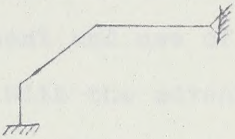
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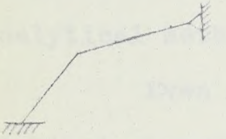
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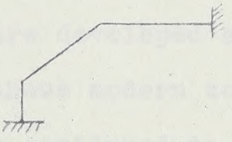
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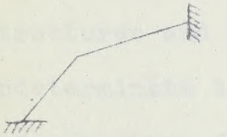
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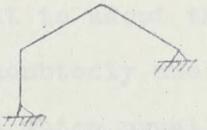
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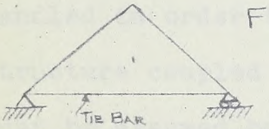

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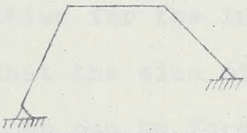
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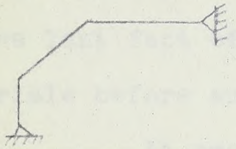
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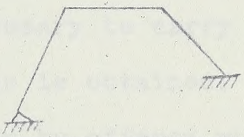
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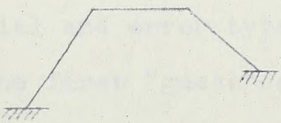
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INTRODUCTION

Throughout the years, lengthy computations have posed as a hurdle to the advancement of structural analysis and indeed to all science. This difficulty was overcome in previous years by the development and use of graphical methods of structural analysis. With the advent of the slide-rule and electric computing machines more accurate, analytical methods of analysis were developed and adopted.

Even with the help of these modern computing machines, however, Engineers have continued to use simple structures and have been reluctant to adopt the more complex indeterminate types. This is undoubtedly due in part to the large number of elastic equations which usually must be handled in order to obtain a solution for the indeterminate structure coupled with the fact that the size of the members must be assumed before the equations can be formulated. The last fact often makes it necessary to carry out several trials before an acceptable design is obtained.

At present no method can be offered which would eliminate this trial and error type of solution and thus the accuracy of the first "guess" and hence, the number of trials required depends only upon the experience of the designer. However, it is possible to decrease the number of elastic equations which must be treated and hence decrease the amount of work required for each trial, a fact which should add impetus to the use of the indeterminate structure

in the future. The method of decreasing the number of elastic equations consists merely of adopting a statically indeterminate structure rather than the usual simple or statically determinate type as the "primary" structure. This is possible however only if a handbook containing solutions for a large number of rigid frames is available. Such a book is Kleinlogel's "Rigid Frame Formulas."

Kleinlogel could not have thought of this application of his works as many cases of loading essential for the development and advancement of this new idea were omitted from his book and it is the object of this thesis to supply this necessary information.

The basic notation set forth by Kleinlogel has been followed, and also in most cases the coefficients for the individual frames, in order that the books will be compatible and can readily be used together. The two frames for which the coefficients were changed are well marked and the new coefficients supplied so there is little chance of error.

It should be explained why the number of solutions appears to be small. It was decided that rather than solve a large number of very special frames it would be better to solve only the more general cases. This greatly decreased the number of solutions required and still the solutions of the special cases may be determined with relative ease from these general cases.

METHOD OF SOLUTION

The method used in the solution of these frames followed the standard procedure for solving indeterminate structures, i.e. the structure was rendered statically determinate, yet stable, by the removal of reactions and/or insertion of hinges. This removal of reactions and insertion of hinges determines the redundants adopted and, wherever possible, the structure was made determinate in such a way that the redundants corresponded to those used by Kleinlogel. This made it possible to use the same coefficients as were used in the parent book.

The Maxwell-Mohr work equation was used to determine the values of the redundants. The Maxwell-Mohr equation is:

$$\Sigma \bar{Q}_m \delta_m - \Sigma \frac{\bar{N}NL}{AE} - \Sigma \int_0^L \frac{\bar{M}MdL}{EI} - \Sigma \int_0^L \frac{\bar{V}VdL}{GA\beta} - \Sigma \frac{\bar{S}SL}{AE}$$

$$- \Sigma \xi \left[\int_0^L \frac{\bar{M}\Delta t_o}{h} dL + \int_0^L \bar{N} t_o dL \right] = 0$$

As customary only the more important term of this equation i.e. the bending term, was retained. The equation was therefore reduced to:

$$\Sigma \bar{Q}_m \delta_m = \Sigma \int_0^L \frac{\bar{M}MdL}{EI}$$

in which

$\bar{Q}_m$ = Virtual force or couple applied at point m

δ_m = Displacement of point m

$\bar{M}$ = Bending moment due to virtual load or couple at m

M = Bending moment due to actual loading

L = Length of member

I = Moment of Inertia of member

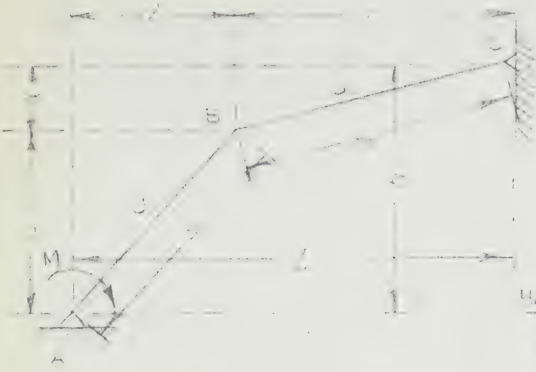
E = Modulus of Elasticity of member

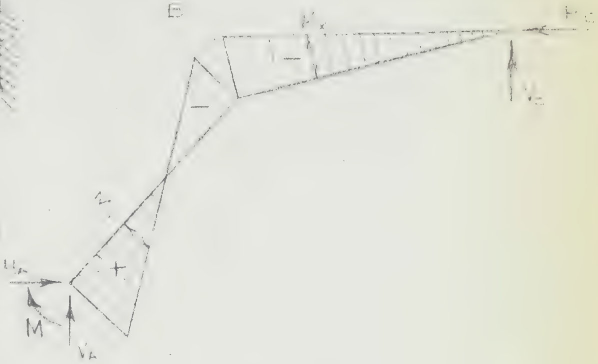
This neglecting of terms could be criticized. However, experience has shown that in most cases of rigid frames the terms neglected are comparatively small being only a few percent of the more important flexure terms. The slight error thus introduced into the design would be adequately cared for by the safety factors applied in present day design procedures.

FOR COEFFICIENTS SEE
KLEINLOGEL PAGE 48

FRAME 15

MOMENT ACTS AT JOINT A



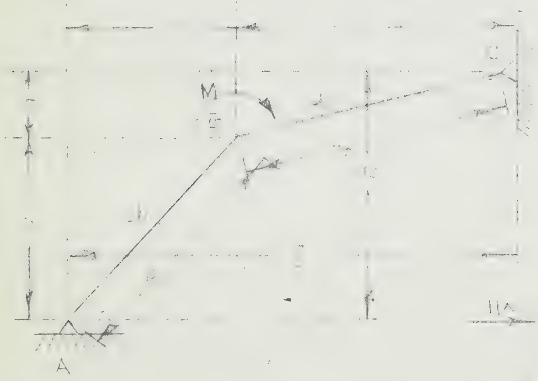


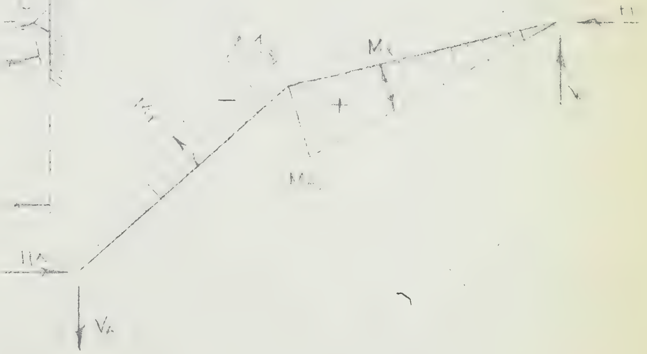
$V_A = -V_c = \frac{M}{F}\left(d + \frac{hk}{2N}\right)$ $H_A = H_c = \frac{1}{h}\left(M + V_c l\right)$

$M_B = dH_c - bV_c$

$M_x = \frac{x'}{b} M_B$ $M_y = M\frac{y'}{c} + M_B\frac{y}{c}$

MOMENT ACTS AT JOINT B





$V_A = V_c = \frac{M}{NF}\left(kc - a\right)$ $H_A = H_c = \frac{1}{h}\left(M - V_c l\right)$

$M_{B1} = -\left(aV_A + cH_A\right)$ $M_{B2} = \left(V_c b + H_c d\right)$

$M_x = \frac{x'}{b} M_{B2}$ $M_y = \frac{y}{c} M_{B1}$

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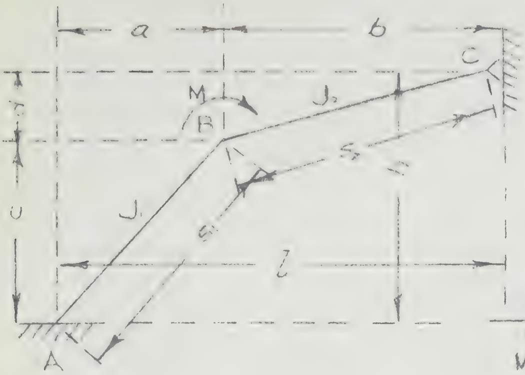
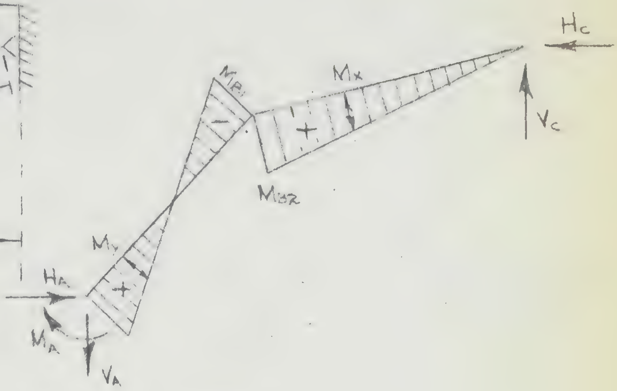
FRAME 16

MOMENT ACTS AT JOINT B





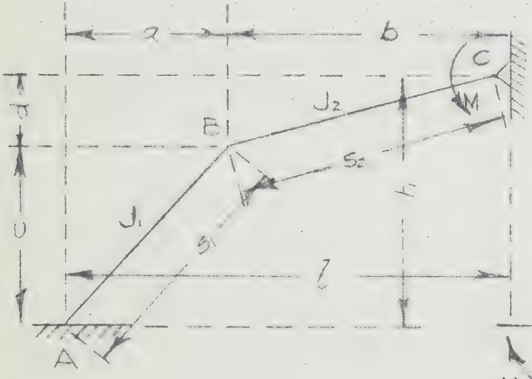
$$V_A = V_C = \frac{3M}{NF}(kc - 2d)$$

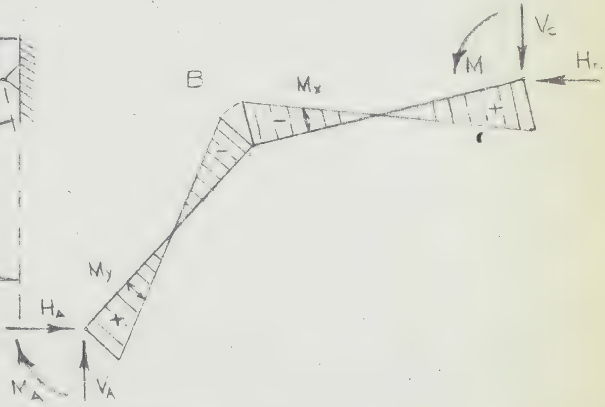
$$H_A = H_C = \frac{3M}{NF}(2b - ka)$$

$$M_A = V_C l + H_C h - M \qquad M_{B1} = M_A - V_A a - H_A c \qquad M_{B2} = V_C b + H_C d$$

$$M_x = M_{B2}\frac{x'}{b} \qquad M_y = M_A\frac{y'}{c} + M_{B1}\frac{y}{c}$$

MOMENT ACTS AT JOINT C





$$V_A = V_C = \frac{3M}{NF}\left[c(k+1) + h\right]$$

$$H_A = H_C = \frac{3M}{NF}\left[a(k+1) + l\right]$$

$$M_A = H_C h + M - V_C l \qquad M_B = M + H_C d - V_C b$$

$$M_x = M_B\frac{x'}{b} + M\frac{x}{b} \qquad M_y = M_A\frac{y'}{c} + M_B\frac{y}{c}$$

FOR COEFFICIENTS SEE
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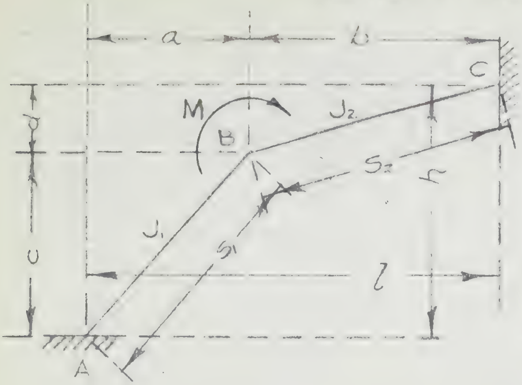
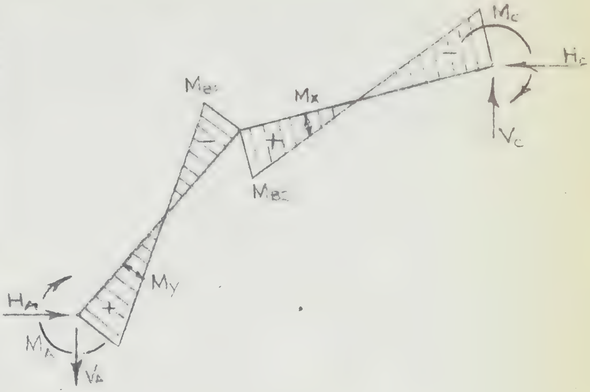
FRAME 18

MOMENT ACTS AT JOINT B





$$M_c = -\frac{kM}{2N} \qquad V_A = V_c = \frac{3M}{2NF}(ck-d) \qquad H_A = H_c = \frac{3M}{2NF}(b-ak)$$

$$M_A = hH_c + lV_c - M_c - M$$

$$M_{B1} = M_A - aV_A - cH_A \qquad M_{B2} = dH_c + bV_c - M_c$$

$$M_x = \frac{x'}{b}M_{B2} + \frac{x}{b}M_c \qquad M_y = \frac{y'}{c}M_A + \frac{y}{c}M_{B1}$$

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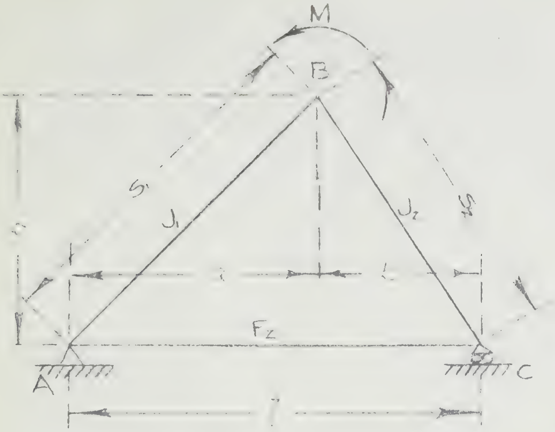
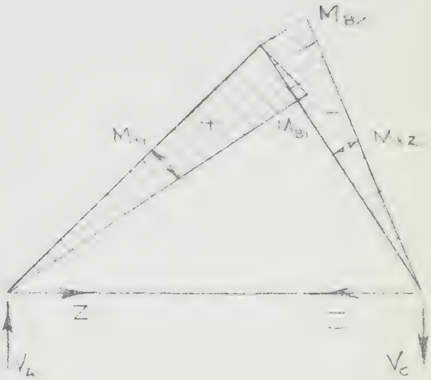
FRAME 23

MOMENT ACTS AT JOINT B





$$V_A = V_C = \frac{M}{l}$$

$$Z = \frac{M}{h N_2}(\alpha - k\beta)$$

$$M_{B1} = M\alpha - Zh$$

$$M_{B2} = -(M\beta + Zh)$$

$$M_{x_1} = \frac{x_1}{a} M_{B1}$$

$$M_{x_2} = \frac{x_2'}{b} M_{B2}$$

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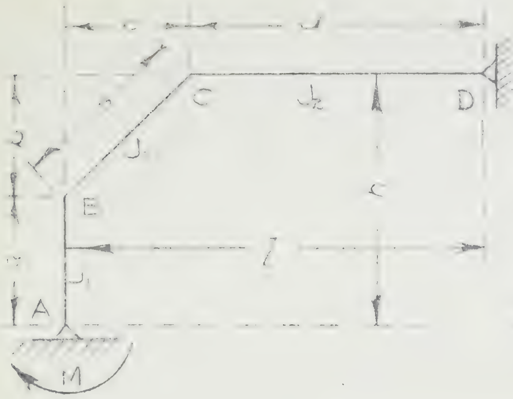
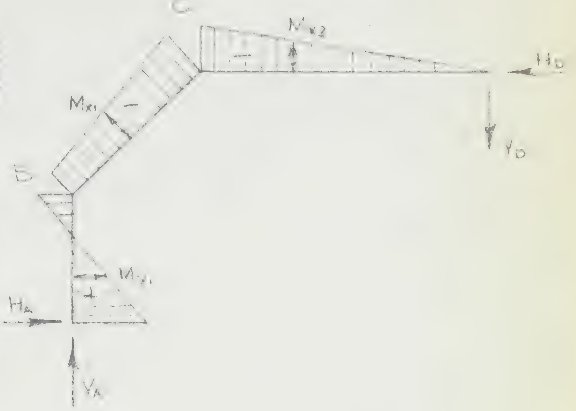
FRAME 34

MOMENT ACTS AT JOINT A



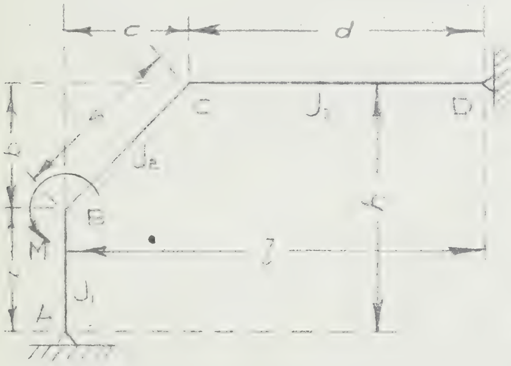


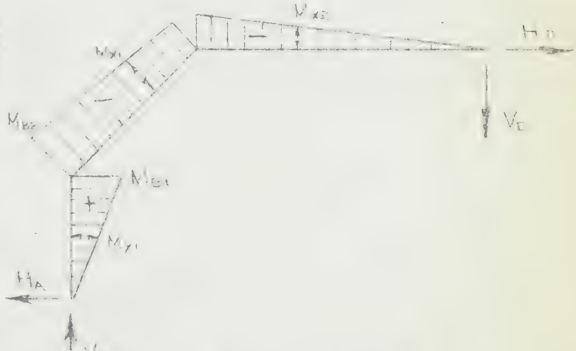
$$V_A = V_D = \frac{M}{Nl}\left(\beta B + k_1\alpha\right) \qquad H_A = H_D = \frac{1}{h}\left(M + V_D l\right)$$

$$M_B = -V_D l + H_D b \qquad M_C = -V_D d$$

$$M_{y1} = \frac{y_1'}{a} M + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C$$

MOMENT ACTS AT JOINT B





$$X = \frac{M}{N}\left[\alpha(k - [illegible]) - [illegible]\right]$$

$$V_A = V_D = \frac{X}{l} \qquad H_A = H_D = \frac{1}{h}\left(M - V_D l\right)$$

$$M_{B1} = a H_A \qquad M_{B2} = -\left(l V_D + b H_D\right) \qquad M_C = -d V_D$$

$$M_{y1} = \frac{y_1}{a} M_{B1} \qquad M_{x1} = \frac{x_1'}{c} M_{B2} + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C$$

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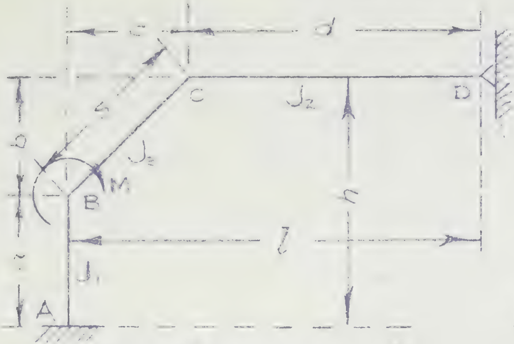
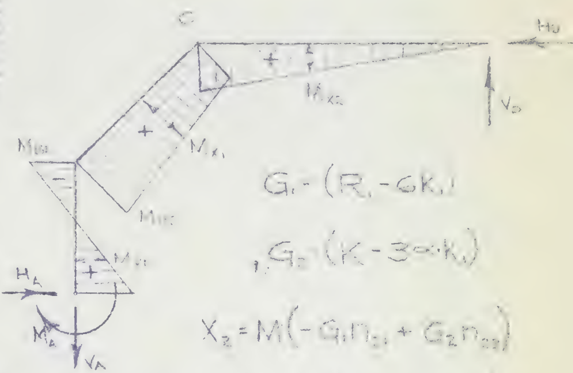
FRAME 35

MOMENT ACTS AT JOINT B





$G_1 = (R_1 - 6K_1)$

$G_2 = (K - 3\alpha K_1)$

$X_1 = M(-G_1 n_{11} + G_2 n_{12})$

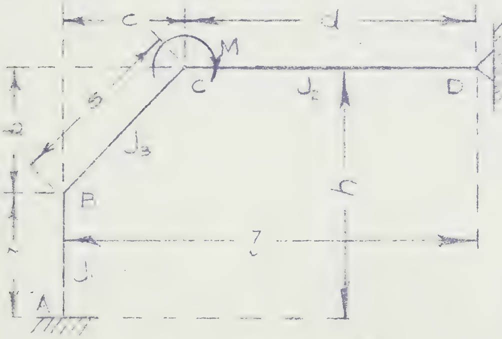
$X_2 = M(-G_1 n_{21} + G_2 n_{22})$

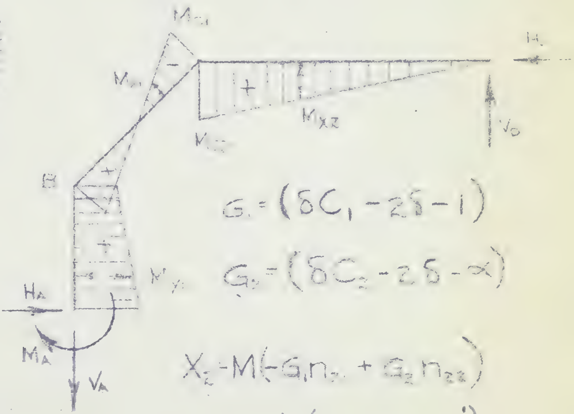
$M_A = X_1$ $\quad H_A = H_D = \frac{X_2}{h}$ $\quad V_A = V_D = \frac{1}{l}(X_1 - X_2 + M)$

$M_{B1} = X_1 - \alpha X_2$ $\quad M_{B2} = X_1 - \alpha X_2 + M$ $\quad M_C = \delta(X_1 - X_2 + M)$

$M_{Y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_{B1}$ $\quad M_{X1} = \frac{x_1'}{c} M_{B2} + \frac{x_1}{c} M_C$ $\quad M_{X2} = M_C \frac{x_2'}{d}$

MOMENT ACTS AT JOINT C





$G_1 = (\delta C_1 - 2\delta - 1)$

$G_2 = (\delta C_2 - 2\delta - \alpha)$

$X_1 = M(-G_1 n_{11} + G_2 n_{12})$

$X_2 = M(-G_1 n_{21} + G_2 n_{22})$

$M_A = X_1$ $\quad H_A = H_D = \frac{X_2}{h}$ $\quad V_A = V_D = \frac{1}{l}(X_1 - X_2 + M)$

$M_B = X_1 - \alpha X_2$ $\quad M_{C1} = \delta(X_1 - X_2) - \gamma M$ $\quad M_{C2} = \delta(X_1 - X_2 + M)$

$M_{Y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B$ $\quad M_{X1} = M_B \frac{x_1'}{c} + \frac{x_1}{c} M_{C1}$ $\quad M_{X2} = \frac{x_2'}{d} M_{C2}$

FOR COEFFICIENTS SEE
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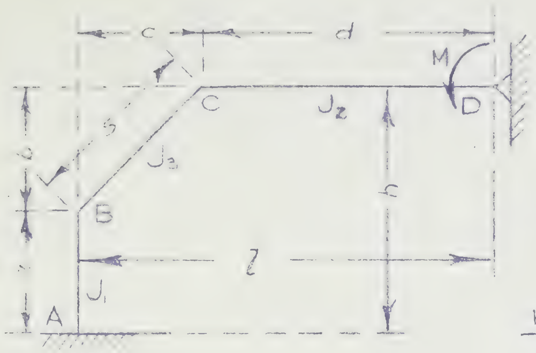
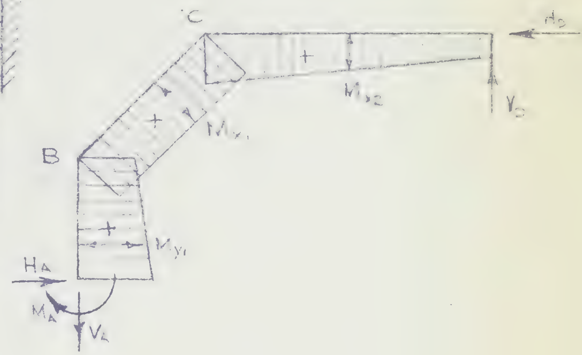
FRAME 35

MOMENT ACTS AT JOINT D





$$G_1 = (\gamma C_1 + \delta k_2) \qquad G_2 = (\gamma C_2 + \delta k_2)$$

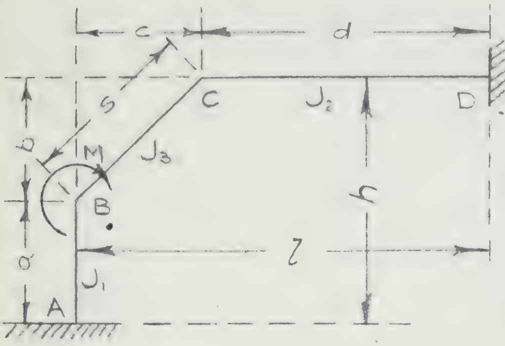
$$X_1 = (-G_1 n_{11} + G_2 n_{12}) \qquad X_2 = (-G_1 n_{21} + G_2 n_{22})$$

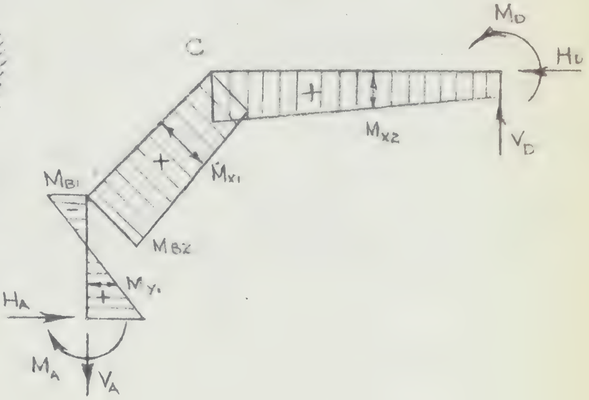
$$M_A = X_1 \qquad H_A = H_D = \frac{X_2}{h} \qquad V_A = V_D = \frac{1}{l}(X_1 - X_2 - M)$$

$$M_B = X_1 - \alpha X_2 \qquad M_C = \delta(X_1 - X_2) + \gamma M$$

$$M_{y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_B \qquad M_{x1} = \frac{x_1'}{c} M_B + \frac{x_1}{c} M_C \qquad M_{x2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M$$

MOMENT ACTS AT JOINT B





$G_1 = (R_1 - 6k_1)$ $\qquad$ $G_2 = K_3$ $\qquad$ $G_3 = (K_2 - 3\alpha k_1)$

$$X_1 = M(-G_1 n_{11} - G_2 n_{12} + G_3 n_{13})$$

$$X_2 = M(-G_1 n_{21} - G_2 n_{22} + G_3 n_{23})$$

$$X_3 = M(-G_1 n_{31} - G_2 n_{32} + G_3 n_{33})$$

$M_A = X_1$ $\qquad$ $M_D = X_2$ $\qquad$ $H_A = H_D = \frac{X_3}{h}$ $\qquad$ $V_A = V_D = \frac{1}{l}(M + X_1 - X_2 - X_3)$

$M_{B1} = X_1 - \alpha X_3$ $\qquad$ $M_{B2} = M + X_1 - \alpha X_3$ $\qquad$ $M_C = \gamma X_2 + \delta(M + X_1 - X_3)$

$M_{Y1} = \frac{y_1'}{a} M_A + \frac{y_1}{a} M_{B1}$ $\qquad$ $M_{X1} = \frac{x_1'}{c} M_{B2} + \frac{x_1}{c} M_C$ $\qquad$ $M_{X2} = \frac{x_2'}{d} M_C + \frac{x_2}{d} M_D$

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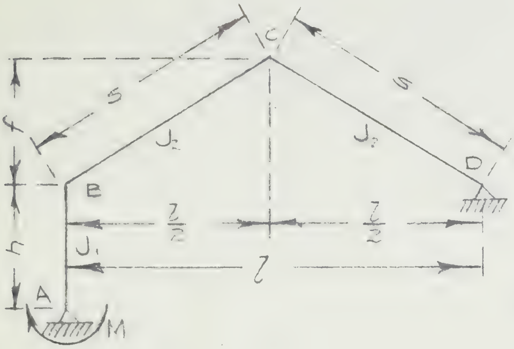
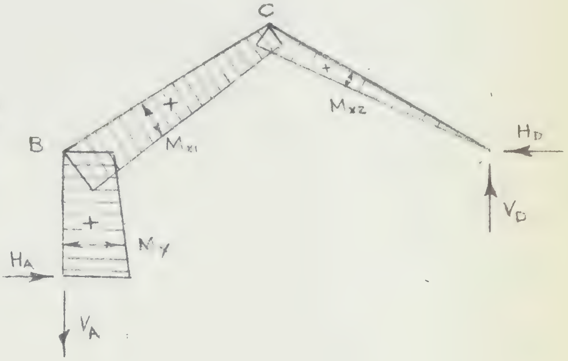
FRAME 38

MOMENT ACTS AT JOINT A



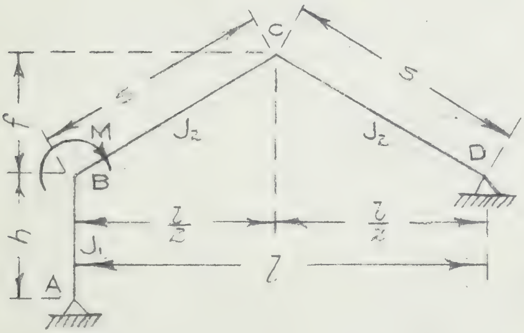


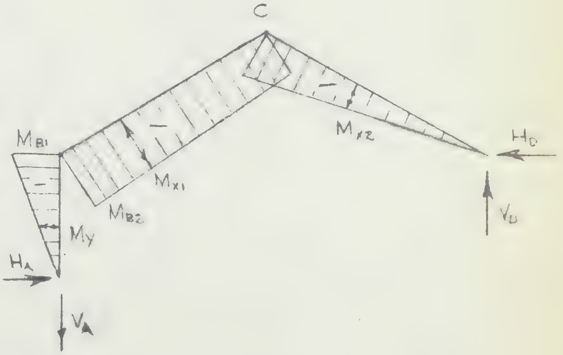
$$H_A = H_D = \frac{M}{Nh}(k+B+C) \qquad V_A = V_D = \frac{1}{l}(M-hH_A)$$

$$M_B = M - hH_A \qquad M_C = \frac{V_D l}{2} - fH_D$$

$$M_y = \frac{y'}{h}M + \frac{y}{h}M_B \qquad M_{x1} = \frac{2x_1'}{l}M_B + \frac{2x_1}{l}M_C \qquad M_{x2} = \frac{2x_2'}{l}M_C$$

MOMENT ACTS AT JOINT B





$$H_A = H_D = \frac{M}{Nh}(4+3\Phi) \qquad V_A = V_D = \frac{1}{l}(M-hH_A)$$

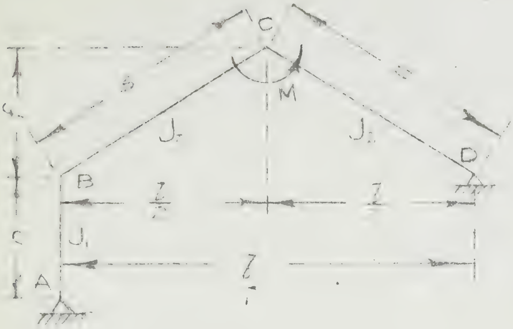
$$M_{B1} = -hH_A \qquad M_{B2} = M - hH_A \qquad M_C = \frac{V_D l}{2} - fH_D$$

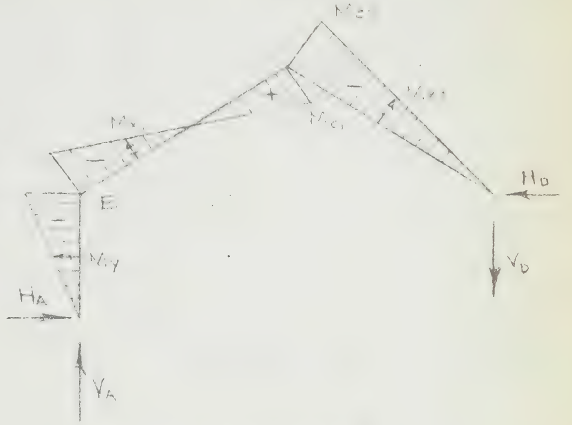
$$M_y = \frac{y}{h}M_{B1} \qquad M_{x1} = \frac{2x_1'}{l}M_{B2} + \frac{2x_1}{l}M_C \qquad M_{x2} = \frac{2x_2'}{l}M_C$$

FOR COEFFICIENTS SEE
[illegible] PAGE 125

FRAME 38

MOMENT ACTS AT JOINT C



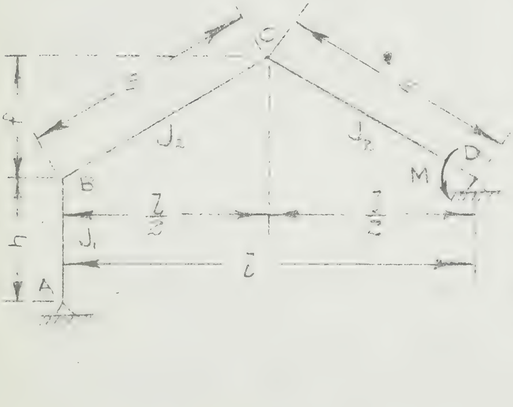


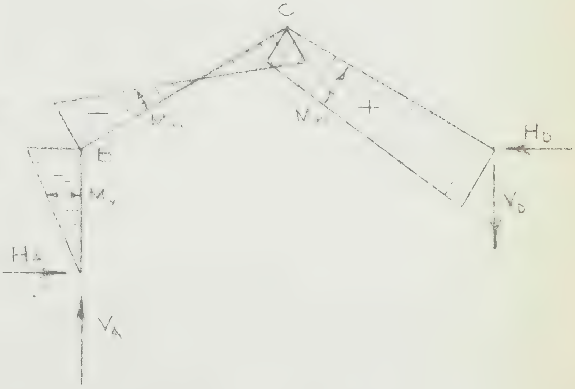
$H_A = H_D = \frac{M}{2Nh}$ $\qquad$ $V_A = V_D = \frac{1}{l}(M + hH_D)$

$M_B = -hH_A$ $\qquad$ $M_{C1} = -H_A(h+f) + V_A\frac{l}{2}$ $\qquad$ $M_{C2} = -\left(\frac{V_D l}{2} + H_D f\right)$

$M_y = \frac{y}{h}M_B$ $\qquad$ $M_x = \frac{2x_1'}{l}M_B + \frac{2x_1}{l}M_{C1}$ $\qquad$ $M_{x2} = \frac{2x_2'}{l}M_{C2}$

MOMENT ACTS AT JOINT D





$H_A = H_D = \frac{M}{Nh}(3\Phi + 2)$ $\qquad$ $V_A = V_D = \frac{1}{l}(M + hH_A)$

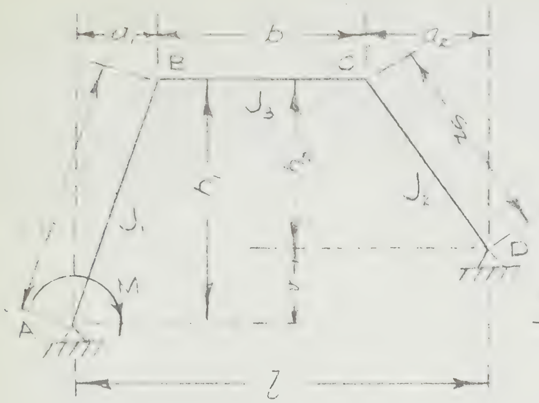
$M_B = -hH_A$ $\qquad$ $M_C = V_A\frac{l}{2} - H_A(h+f)$

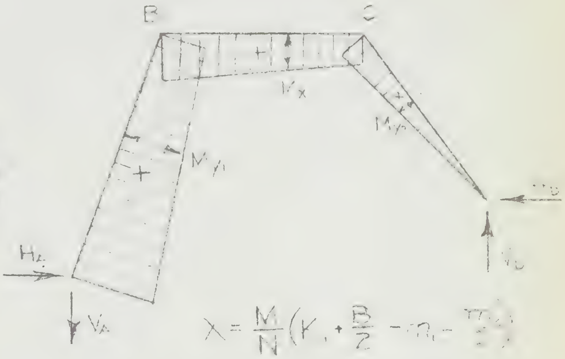
$M_y = M_B\frac{y}{h}$ $\qquad$ $M_{x1} = \frac{2x_1'}{l}M_B + \frac{2x_1}{l}M_C$ $\qquad$ $M_{x2} = \frac{2x_2'}{l}M_C + \frac{2x_2}{l}M$

FOR COEFFICIENTS SEE
KLEINLOGEL PAGE 398

FRAME 75

MOMENT ACTS AT JOINT A





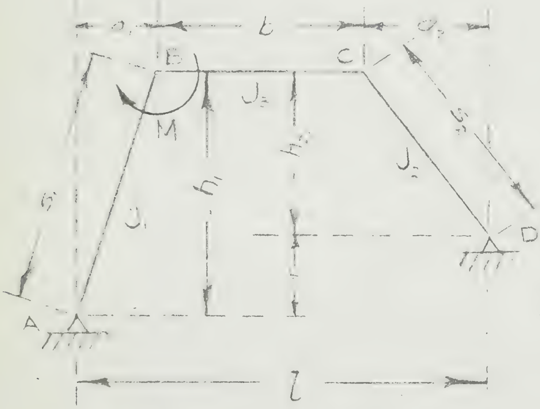
$$X = \frac{M}{N}\left(K_1 + \frac{B}{2} - m_1 - \frac{m_2}{2}\right)$$

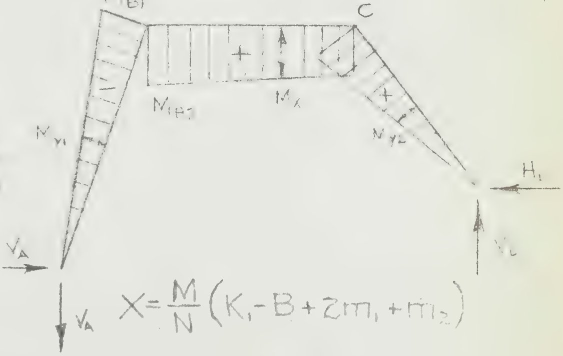
$$H_A = H_D = \frac{X}{h} \qquad V_A = V_D = \frac{1}{l}\left[M - (1-n)X\right]$$

$$M_B = \beta_1(M-X) - \alpha_1 n X \qquad M_C = \alpha_2(M-X) - \beta_2 n X$$

$$M_{y_1} = \frac{y_1'}{h_1} M + \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_B + \frac{x}{b} M_C \qquad M_{y_2} = \frac{y_2}{h_2} M_C$$

MOMENT ACTS AT JOINT B





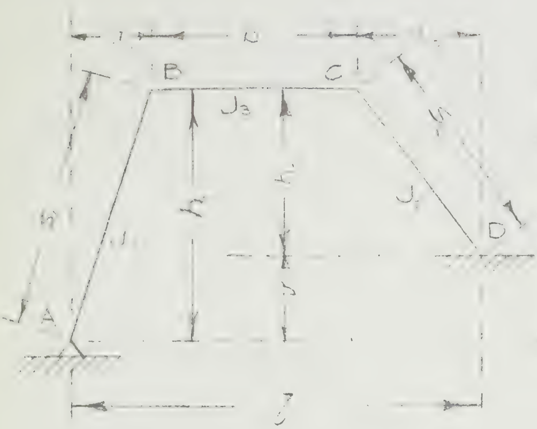
$$X = \frac{M}{N}\left(K_1 - B + 2m_1 + m_2\right)$$

$$H_A = H_D = \frac{X}{h_1} \qquad V_A = V_D = \frac{1}{l}\left[M - (1-n)X\right] \qquad M_{B_1} = -\varepsilon_1 X - \alpha_1(M - nX)$$

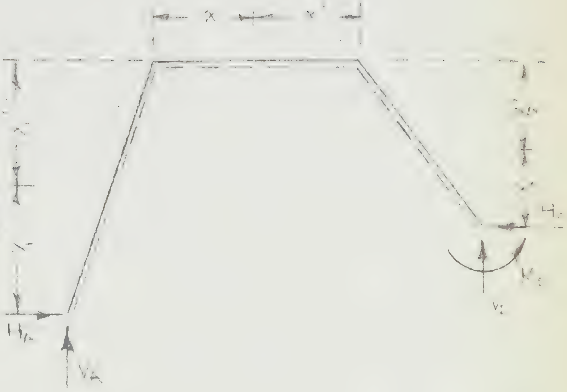
$$M_{B2} = \beta_1(M-X) - n\alpha_1 X \qquad M_C = \alpha_2(M-X) - \beta_2 n X$$

$$M_{y_1} = \frac{y_1}{h_1} M_B \qquad M_x = \frac{x'}{b} M_{B2} + \frac{x}{b} M_C \qquad M_{y_2} = \frac{y_2}{h_2} M_C$$

FRAME 76



DIMENSIONS & NOTATION



POSITIVE DIRECTION REACTIONS
COORDINATES OF ANY POINT.
POSITIVE BENDING CAUSES TENSION
ON FACE MARKED BY DASHED
LINE

COEFFICIENTS

$$k_1 = \frac{J_3}{J_1} \cdot \frac{s_1}{b} \qquad k_2 = \frac{J_3}{J_2} \cdot \frac{s_2}{b} \qquad n = \frac{h_2}{h_1} \qquad \alpha_1 = \frac{a_1}{b} \qquad \alpha_2 = \frac{a_2}{b}$$

$$\delta = \alpha_1 + 1 \qquad \beta = \delta n + \alpha_2 \qquad \gamma = n\alpha_1 + 1 + \alpha_2 \qquad D = (1+2\gamma)k_2$$

$$R_1 = \left[2k_1\alpha_1^2 + 2k_2\left(3\delta^2 + 3\alpha_2\delta + \alpha_2^2\right) + 2\left(1 + 3\text{[illegible]}\delta\right)\right]$$

$$R_2 = \left[2k_1 + 2k_2(3 - 3n + n^2) + 6\right]$$

$$K = \left[2k_1\alpha_1 + k_2\left\{3\delta(2-n) + \alpha_2(3-2n)\right\} + 3(\delta + \alpha_1)\right]$$

$$N = R_1 R_2 - K^2$$

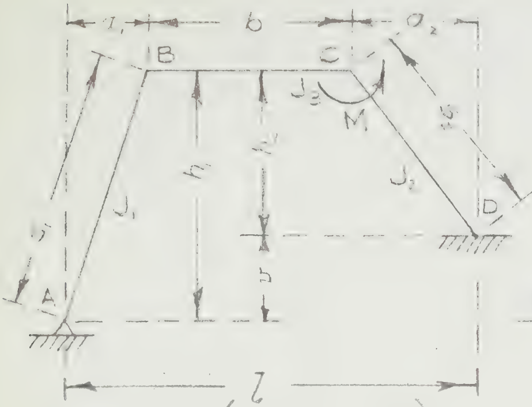
$$n_{11} = \frac{R_2}{N} \qquad n_{12} = n_{21} = \frac{K}{N} \qquad n_{22} = \frac{R_1}{N}$$

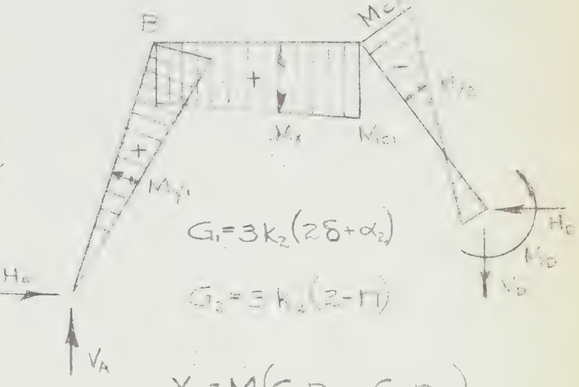
NOTE THESE COEFFICIENTS ARE NOT THE SAME AS THOSE IN KLEINLOGEL

FOR COEFFICIENTS SEE PAGE 12 IN THE BOOK

FRAME 76

MOMENT ACTS AT JOINT C





$G_1 = 3k_2(2\delta + \alpha_2)$

$G_2 = 3k_2(2-n)$

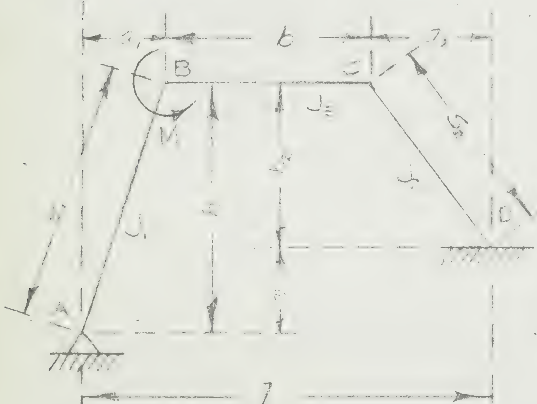
$X_1 = M(G_1 n_{11} - G_2 n_{12})$ $X_2 = M(G_1 n_{21} - G_2 n_{22})$

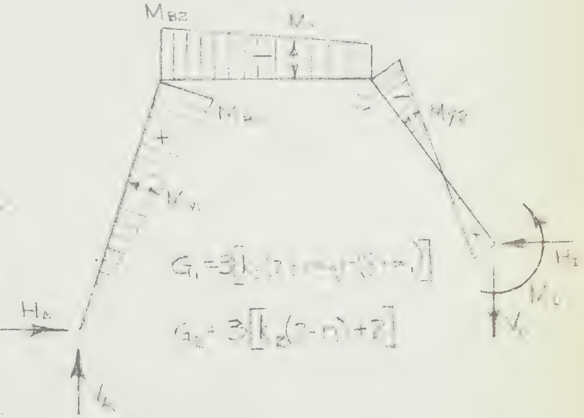
$H_A = H_D = \frac{X_2}{h_1}$ $V_A = V_D = \frac{X_1}{b}$ $M_B = \alpha_1 X_1 - X_2$

$M_{C1} = \delta X_1 - X_2$ $M_{C2} = \delta X_1 - X_2 - M$ $M_D = X_1 \frac{l}{b} - X_2(1-n) - M$

$M_{y1} = \frac{y_1}{h_1} M_B$ $M_x = \frac{x'}{b} M_B + \frac{x}{b} M_{C1}$ $M_{y2} = \frac{y_2'}{h_2} M_{C1} + \frac{y_2}{h_2} M_D$

MOMENT ACTS AT JOINT B





$G_1 = 3[k_2(1+n) + (\delta + \alpha_1)]$

$G_2 = 3[k_2(2-n) + 2]$

$X_1 = M(G_1 n_{11} - G_2 n_{12})$ $X_2 = (G_1 n_{21} - G_2 n_{22})$

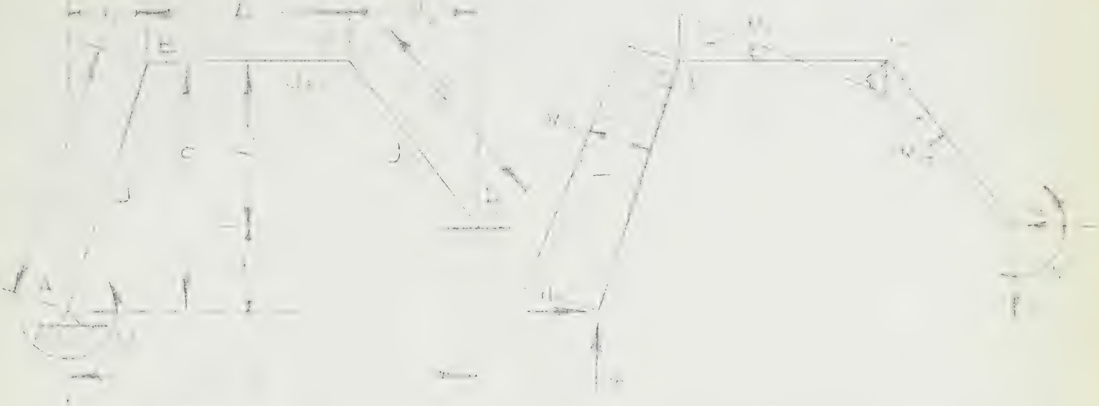
$H_A = H_D = \frac{X_2}{h_1}$ $V_A = V_D = \frac{X_1}{b}$ $M_{B1} = \alpha_1 X_1 - X_2$

$M_{B2} = \alpha_1 X_1 - X_2 - M$ $M_C = \delta X_1 - X_2 - M$ $M_D = X_1 \frac{l}{b} - X_2(1-n) - M$

$M_{y1} = \frac{y_1}{h_1} M_{B1}$ $M_x = \frac{x'}{b} M_{B2} + M_C \frac{x}{b}$ $M_{y2} = \frac{y_2}{h_2} M_C + \frac{y_2}{h_2} M_D$

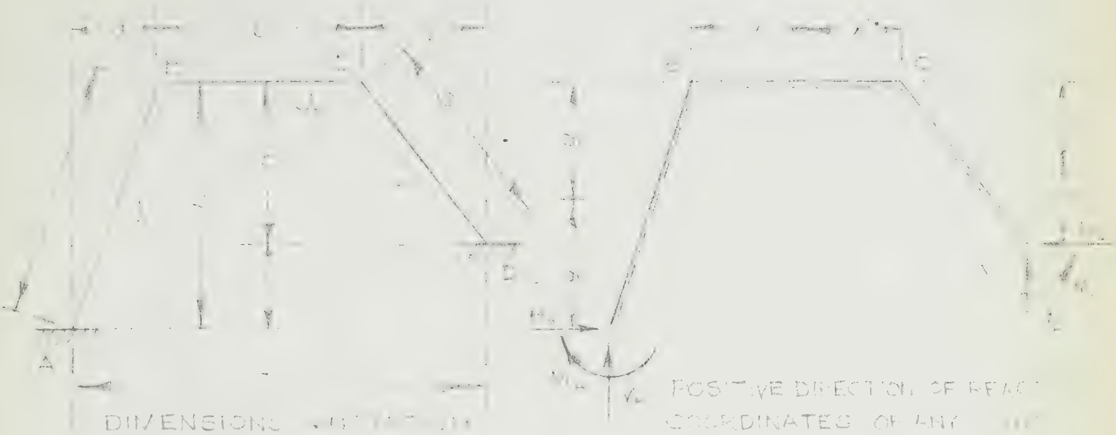
[illegible]

MOMENT ACTS AT JOINT A



[illegible]

FRAME 78



POSITIVE DIRECTION OF REACTIONS
COORDINATES OF ANY POINT
POSITIVE BENDING MOMENT:
TENSION ON FACE MARKED
BY DASHED LINE.

COEFFICIENTS *

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

---

* NOTE [illegible]

[illegible]

[illegible]

$X = M (G_1 n_{11} +$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

M [illegible]

M [illegible] M_1 [illegible] M [illegible] M_1

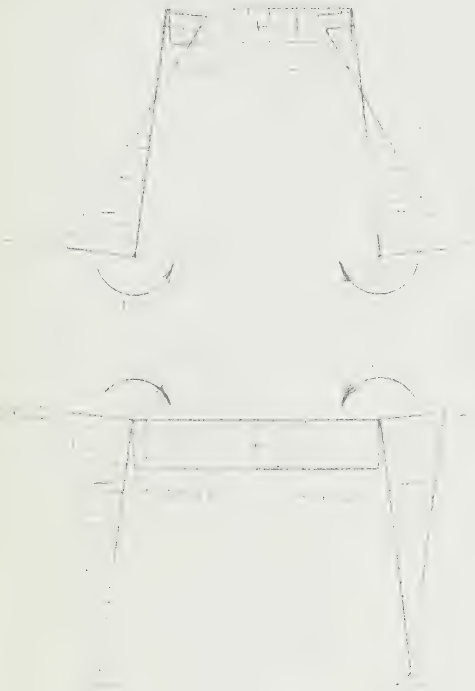
[illegible]

[illegible]

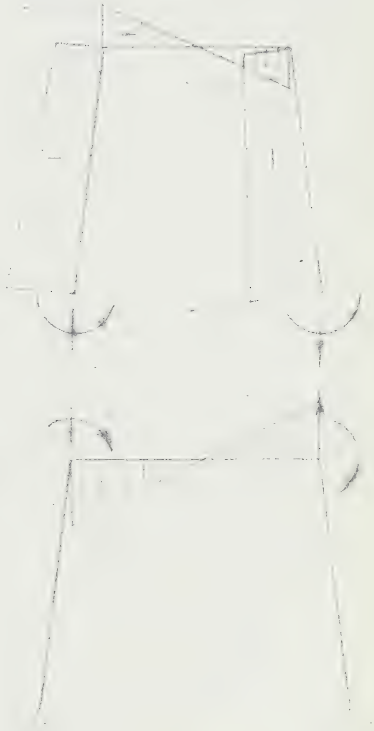
[illegible]
Charges at [illegible]
Foundations are
[illegible]

[illegible]

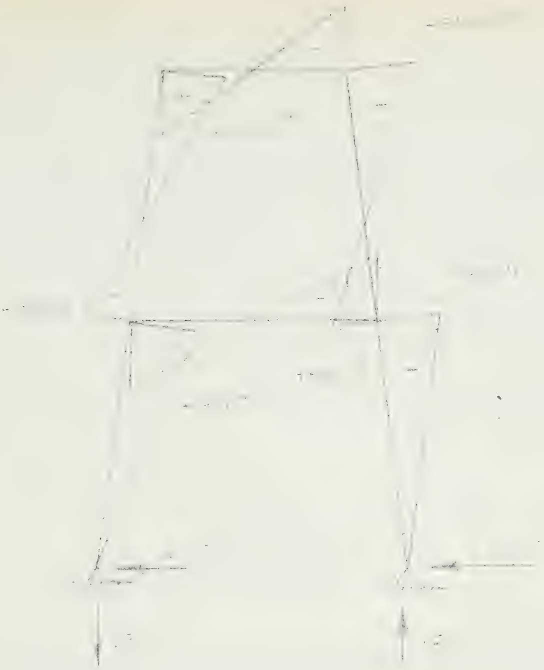
[illegible]



[illegible]



[illegible]



[illegible]

ACKNOWLEDGMENT

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3. Morrison, I. F. -- On the extended use of Kleinlogels Rachmenformalen, Canadian Journal of Research, December, 1948.
4. Müller Breslau -- Die Graphesche Statik der Baukonstruktionen, Volume 2, 1924.